Eur. Phys. J. C 4, 497–502 (1998)
DOI 10.1007/s100529800777

THE EUROPEAN
PHYSICAL JOURNAL C

Photon polarisation in diffractive deep inelastic scattering

M. Diehl

CPT, Ecole Polytechnique, F-91128 Palaiseau, France

Received: 17 September 1997 / Published online: 23 February 1998

Abstract. The distribution of a suitably defined azimuthal angle in diffractive deep inelastic scattering contains information on the polarisation of the exchanged photon. In particular it allows one to constrain the longitudinal diffractive structure function. We investigate the potential of such bounds in general and for particular diffractive final states.

1 Introduction

The inclusive cross section for deep inelastic diffraction measured at HERA [1] shows a remarkable pattern of scaling violation: the diffractive structure function $F_2^{D(3)}(x_{I\!P}, \beta, Q^2)$ is found to rise with Q^2 even at rather large values of the scaling variable β. When F_2^D is interpreted in terms of diffractive parton densities evolving according to the DGLAP equations this leads to a significant amount of gluons with a large momentum fraction. An important question for the QCD analysis of F_2^D and also for its extraction from the data is how much of F_2^D is due to longitudinally polarised photons. Several models of diffraction find in fact a considerable longitudinal contribution F_L^D to $F_2^D = F_T^D + F_L^D$ at large β [2]. It is of course crucial to know whether or not such a contribution is of leading twist if one wants to describe F_2^D in terms of leading twist parton densities and their evolution [3].

In [4,5] its was pointed out that an appropriate azimuthal distribution in the final state can be used to obtain bounds on F_L^D, without requiring measurements at different energies of the ep collision as in the standard method for the separation of longitudinal and transverse structure functions. The aim of this paper is to make some comments on the potential of these bounds in general, and to see what can be expected for F_L^D and its bounds for particular diffractive final states and dynamical models.

2 The azimuthal angle

Let us consider a diffractive reaction $e(k) + p(p) \to e(k') + X(p_X) + \tilde{p}(\tilde{p})$, where X is the diffractive system and $\tilde{p}$ the scattered proton or proton remnant and where we have indicated four-momenta in parentheses. We will always work in the one-photon exchange approximation. If we define some four-vector τ in the final state and go to the $\gamma^* p$ c.m. with the positive z axis defined by the photon momentum q then we have an azimuthal angle φ between the electron momentum k and τ (Fig. 1) which contains information on the polarisation of the exchanged photon. For τ we have the freedom of choice under the condition [4] that it should only depend on momenta of the subreaction $\gamma^*(q) + p(p) \to X(p_X) + \tilde{p}(\tilde{p})$. Here we choose the following: go to the rest frame of the system X and set $\tau = (0, \vec{\tau})$ where $\vec{\tau}$ is the thrust axis of X oriented to point into the photon direction. If X consists only of two particles then $\vec{\tau}$ simply is the direction of the forward particle as shown in Fig. 2 (a), the general case is represented in Fig. 2 (b).

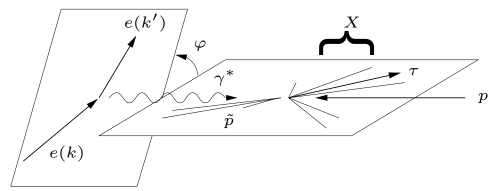

Fig. 1. Kinematics of a diffractive process in the $\gamma^* p$ c.m. The vector τ is defined in Fig. 2

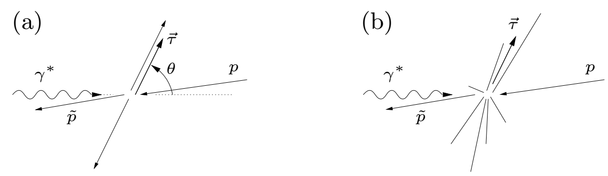

Fig. 2. Definition of $\vec{\tau}$ as the thrust axis in the c.m. of the diffractive system X, oriented to point into the photon direction. θ is the angle between $\vec{\tau}$ and the photon momentum

The dependence of the ep cross section on this angle is explicitly given as a trigonometric polynomial [4,5]

$$\begin{aligned}\frac{d\sigma(ep \to e\tilde{p}X)}{d\varphi\, dQ^2\, dx\, dx_{I\!P}\, d\Phi} &= \frac{\alpha_{em}}{2\pi^2}\, \frac{1-x}{xQ^2}\, \left(1 - y + y^2/2\right) \\ &\cdot \big\{ S_{++} + \varepsilon S_{00} - \varepsilon S_{+-} \cdot \cos 2\varphi \\ &-2\sqrt{\varepsilon(1+\varepsilon)}\, \mathrm{Re} S_{+0} \cdot \cos\varphi \\ &+2 r_L \sqrt{\varepsilon(1-\varepsilon)}\, \mathrm{Im} S_{+0} \cdot \sin\varphi \big\} \quad , \qquad (1)\end{aligned}$$

where $r_L = \pm 1$ is the helicity of the incident lepton. We have used the conventional variables $Q^2 = -q^2$, $x = Q^2/(2q \cdot p)$, $y = (q \cdot p)/(k \cdot p)$, $\beta = Q^2/(2q \cdot \Delta)$, $x_{I\!P} = (q \cdot \Delta)/(q \cdot p)$ with $\Delta = p - \tilde{p}$, and the usual ratio $\varepsilon = (1-y)/(1-y+y^2/2)$ of longitudinal and transverse photon flux. The functions

$$S_{mn}(x_{I\!P}, \beta, Q^2, \Phi) = \frac{d\sigma_{mn}}{dx_{I\!P}\, d\Phi} \ , \qquad m,n = -,0,+ \quad (2)$$

do *not* depend on φ, for $m = n$ they are the differential $\gamma^* p$ cross sections for photon helicity m, and for $m \neq n$ they give the interference between photon helicities m and n. With Φ we have denoted any additional variables of the $\gamma^* p \to X\tilde{p}$ reaction one may want to consider, provided that they are invariant under a parity transformation, which excludes e.g. further azimuthal angles. S_{+-} is real under these circumstances whereas S_{+0} may have an imaginary part, which one can however expect to be small compared with its real part [5]. Note that for the appearance of S_{+0} in the ep cross section it is essential that φ is a genuine azimuthal angle ranging from 0 to 2π; if one were to define φ as the angle between the two planes shown in Fig. 1 then φ and $\varphi + \pi$ would be equivalent and terms with $\cos\varphi$ and $\sin\varphi$ would average out in (1).

3 Bounds on the longitudinal cross section

From the φ dependence of the ep cross section one obtains the interference terms S_{+-} and S_{+0} in addition to the weighted sum $S_\varepsilon = S_{++} + \varepsilon S_{00}$ of $\gamma^* p$ cross sections. These allow to constrain S_{00} as [5]

$$\frac{S_\varepsilon - S_{+-}}{2\varepsilon} - \sqrt{\left(\frac{S_\varepsilon - S_{+-}}{2\varepsilon}\right)^2 - \frac{2\,|S_{+0}|^2}{\varepsilon}} \ \leq\ S_{00} \ , \quad (3)$$

$$S_{00} \ \leq\ \frac{S_\varepsilon - S_{+-}}{2\varepsilon} + \sqrt{\left(\frac{S_\varepsilon - S_{+-}}{2\varepsilon}\right)^2 - \frac{2\,|S_{+0}|^2}{\varepsilon}} \ , \quad (4)$$

$$S_{00} \ \leq\ \frac{S_\varepsilon + S_{+-}}{\varepsilon} \ . \quad (5)$$

If $\mathrm{Im} S_{+0}$ is unknown because the lepton beam is unpolarised one can replace S_{+0} with $\mathrm{Re} S_{+0}$ here and in the sequel. Weaker but simpler versions of bounds (3) and (4) are

$$\frac{2\,|S_{+0}|^2}{S_\varepsilon - S_{+-}} \ \leq\ S_{00} \ , \quad (6)$$

$$S_{00} \ \leq\ \frac{S_\varepsilon - S_{+-}}{\varepsilon} \ , \quad (7)$$

respectively, they correspond to the leading terms when (3) and (4) are Taylor expanded in $|S_{+0}|^2/(S_\varepsilon - S_{+-})^2$.

To obtain bounds on the Φ-integrated longitudinal cross section $d\sigma_{00}/dx_{I\!P}$, from which the diffractive structure function $F_L^{D(3)}(x_{I\!P}, \beta, Q^2)$ is obtained by multiplying with $Q^2 \cdot (1-x)/(4\pi^2 \alpha_{em})$, there are two possibilities:

1. one can extract S_ε, S_{+-}, S_{+0} differential in certain variables Φ, evaluate the bounds (3) to (7) on S_{00} and then integrate over Φ, or
2. one can determine S_ε, S_{+-}, S_{+0} already integrated over Φ and evaluate (3) to (7).

For bounds (5) and (7) the two procedures are obviously equivalent, but for the other ones they are not. While the second possibility allows for a more inclusive measurement it is an easy exercise to show that bounds (3), (4) and (6) become weaker each time one integrates over a variable before evaluating them, except if $S_{+0}/(S_\varepsilon - S_{+-})$ is constant in that variable—in this case procedures 1. and 2. give again the same result. In practice this means that if $S_\varepsilon - S_{+-}$ and S_{+0} have a quite different behaviour in a variable Φ one can expect the bounds (3), (4), (6) to be better if they are first evaluated with some binning in Φ and then integrated. An example of such a variable is the polar angle θ of the thrust axis defined in Fig. 2 (a), as we shall see.

We now show that there is a limit on how good the bounds (3), (4), (5) can be. For this we notice that there is an upper bound on the interference terms between longitudinal and transverse photons:

$$2\,|S_{+0}|^2 \leq S_{00}\,(S_{++} - S_{+-}) \ . \quad (8)$$

To see this it is convenient to change basis from circular to linear photon polarisation vectors, related by $\varepsilon_+ = -(\varepsilon_1 + i\varepsilon_2)/\sqrt{2}$ and $\varepsilon_- = (\varepsilon_1 - i\varepsilon_2)/\sqrt{2}$ where ε_1 lies in the plane spanned by τ and q in the $\gamma^* p$ frame. With the constrains from parity invariance [4] one has $S_{10} = -\sqrt{2}\, S_{+0}$, $S_{11} = S_{++} - S_{+-}$, $S_{22} = S_{++} + S_{+-}$ so that the above inequality reads

$$|S_{10}|^2 \leq S_{00}\, S_{11} \ . \quad (9)$$

Now we use that up to a flux and phase space factor S_{mn} is given by $\int d\Phi'\, \mathcal{A}_m^* \mathcal{A}_n$ where $\mathcal{A}_m$ is the amplitude of $\gamma^* p \to X\tilde{p}$ for photon polarisation m and Φ' denotes all variables over which S_{mn} is already integrated. We include in Φ' the polarisations of p and the final state particles, for which the integral reduces to a sum. Taking the functions $\mathcal{A}_m(\Phi')$ as elements of a Hilbert space and the integral over Φ' as a scalar product (9) is just the Schwarz inequality.

This argument also tells us that we have equality in (8), (9) exactly if $\mathcal{A}_0$ and $\mathcal{A}_1$ are proportional to each other as functions of Φ'. In this case the l.h.s. of (3) and the r.h.s. of (4) reduce to $\frac{1}{2\varepsilon}(S_{++} - S_{+-} + \varepsilon S_{00}) \pm \frac{1}{2\varepsilon}|S_{++} - S_{+-} - \varepsilon S_{00}|$, i.e. to

$$S_{00} \qquad \text{and} \qquad \frac{S_{++} - S_{+-}}{\varepsilon} \ , \quad (10)$$

so that one of the bounds on S_{00} in (3), (4) becomes equal to S_{00} itself, this can be the lower or the upper bound. If (8) is a strict inequality then the bounds (3), (4) are less good than in (10). We see that they are rather far apart if S_{00} is much smaller or much bigger than $S_{++} - S_{+-} = S_{11}$. For (3) and (4) to be tight bounds one needs a region of phase space where the $\gamma^* p$ scattering amplitudes with transverse and longitudinal photons are of comparable magnitude and where they have a large enough interference.

As to the upper bound (5) one easily sees that it equals S_{00} if $S_{++} + S_{+-} = S_{22} = 0$ and is bigger otherwise.

4 Particular diffractive final states

4.1 Two spin zero mesons

To see that the above "optimal bounds" on S_{00} can actually be achieved in realistic cases let us consider a very simple diffractive system $X = M\bar{M}$, where M stands for a spin zero meson such as a pion or kaon, and let $\tilde{p}$ be an elastically scattered proton. In $S_{mn} = d\sigma_{mn}/(dx_{I\!P}\, d\cos\theta)$ several degrees of freedom Φ' are summed or integrated over:

1. the solid angle of the scattered proton in the $\gamma^* p$ c.m. In the diffractive region it is a good approximation to replace this integration with taking the $\gamma^* p$ cross sections and interference terms at zero scattering angle and multiplying with a common overall factor. In the following we therefore consider the incoming and outgoing proton to be collinear in the $\gamma^* p$ c.m.
2. the two configurations where the forward particle is M or $\bar{M}$, related by swapping the meson momenta. Assuming that the $\gamma^* p$ reaction can be described by exchanges of positive charge conjugation parity between p and γ^*, which of course holds if pomeron exchange dominates, it follows from charge conjugation invariance of the subreaction $\gamma^* + \text{(exchange)} \to M\bar{M}$ that the $\gamma^* p$ cross sections and interference terms are equal for these configurations, cf. [5].
3. the helicities h and $\tilde{h}$ of initial and scattered proton in the $\gamma^* p$ frame; for zero scattering angle they are the same in the rest frame of X. We make the assumption that the $\gamma^* p$ amplitudes $\mathcal{A}_m^{h,\tilde{h}}$ satisfy $\mathcal{A}_m^{++} = \mathcal{A}_m^{--}$ and $\mathcal{A}_m^{+-} = \mathcal{A}_m^{-+} = 0$. This holds for instance in the limit of large $\gamma^* p$ c.m. energy in the two-gluon exchange model of Landshoff and Nachtmann [6] and in the pomeron model of Donnachie and Landshoff [7].

With these approximations the condition needed for (8) to be an equality are satisfied and the bounds (3), (4) take the form (10). One can say more: at $\theta = 0$ it follows from angular momentum conservation that the γ^* must be longitudinal and $S_{++} - S_{+-} = 0$. At $\theta = \pi/2$ it is S_{00} that must vanish: a rotation by π about the z axis followed by charge conjugation of $\gamma^* + \text{(exchange)} \to M\bar{M}$ gives $\mathcal{A}_0^{++} = -\mathcal{A}_0^{++}$ under our assumptions, the origin of the minus sign being the negative charge conjugation parity of the photon. Assuming that S_{00} is not also zero at $\theta = 0$ or $S_{++} - S_{+-}$ at $\theta = \pi/2$ we then have that S_{00} in (10) is the upper bound for θ near 0 and the lower one for θ near $\pi/2$, and at some value of θ the curves for the two bounds in (10) will cross over.

In the X rest frame a parity transformation and subsequent rotation by π about the axis perpendicular to the scattering plane gives $\mathcal{A}_2^{++} = -\mathcal{A}_2^{--}$, where the minus sign comes from the transformation of the photon polarisation ε_2. With $\mathcal{A}_2^{--} = \mathcal{A}_2^{++}$ we thus have $S_{++} + S_{+-} = S_{22} = 0$ with our assumptions.[1] Hence bound (5) is the longitudinal cross section itself. For θ near $\pi/2$ where S_{00} is smaller than $(S_{++} - S_{+-})/\varepsilon = 2S_{++}/\varepsilon$ one has that both a lower (3) and an upper (5) bound are equal to S_{00} which then is completely constrained. Our assumptions in points 1. to 3. will of course not be exactly satisfied but one can expect that very close bounds on the longitudinal cross section can be obtained for $X = M\bar{M}$ final states.

[1] This result still holds if instead of being zero the amplitudes $\mathcal{A}_m^{+-}$ and $\mathcal{A}_m^{-+}$ have equal size and an appropriate relative phase

4.2 $X = q\bar{q}$

We now look at the diffractive final state at parton level, where calculations have been made in several models of diffraction. The simplest state is a quark-antiquark pair, for which detailed predictions including the $\gamma^* p$ interference terms are available in two-gluon exchange models [5, 8]. We will first consider light quark flavours and neglect the quark mass.

If the quark and antiquark are only produced with opposite helicities, which is the case for massless quarks in the two-gluon models cited, and if one makes the same assumptions on the scattering of the proton as in points 1. to 3. of the previous subsection, one finds again that (8) is an equality. Compared with $M\bar{M}$ one now has an additional summation in $S_{mn} = d\sigma_{mn}/(dx_{I\!P}\, d\cos\theta)$ over the two $q\bar{q}$ helicity combinations. Working in the c.m. of X one can relate the corresponding amplitudes by a parity transformation followed by rotation of π about the axis perpendicular to the scattering plane and finds that $\mathcal{A}_0$ and $\mathcal{A}_1$ are again proportional as functions of Φ'.

Let us recall some results for the dependence of the $\gamma^* p$ cross sections and interference terms on Q^2 and on the transverse momentum P_T of the produced quark in the $\gamma^* p$ c.m., given by $\sin\theta = 2P_T/M$ where M is the invariant $q\bar{q}$ mass. If P_T/M is small then S_{00} and S_{+-} are suppressed by a factor P_T^2/M^2 and S_{+0} by a factor P_T/M compared to S_{++} which dominates in this region, while at large P_T all terms can be of comparable magnitude. S_{++} approximately falls like $1/P_T^4$ in the range $1\ \text{GeV}^2 \lesssim P_T^2 \ll M^2$. The P_T-integrated transverse cross section $d\sigma_{++}/dx_{I\!P}$ is dominated by small P_T and behaves like $1/Q^2$ at fixed $x_{I\!P}$ and β, which means Bjorken scaling of F_T^D. In contrast to this the leading power is $1/Q^3$ for $d\sigma_{+0}/dx_{I\!P}$ and $1/Q^4$ for $d\sigma_{00}/dx_{I\!P}$ and $d\sigma_{+-}/dx_{I\!P}$ so that in particular F_L^D is of higher twist. Note however that F_T^D vanishes like $1-\beta$ in the limit $\beta \to 1$ whereas F_L^D is finite, so that in a region of sufficiently large β and not too large Q^2 the longitudinal structure function F_L^D can be appreciable. In such a region, where its role is most important, $c\bar{c}$ production is suppressed or zero due to its production threshold, which justifies the restriction of our discussion to light flavours. Independent of the quark mass one finds that S_{+-} is positive whereas S_{+0} changes sign at some value of β below 1/2, being positive below and negative above.

In Fig. 3 we show an example of the θ dependence of the differential bounds (3), (4) given by (10) and of their weaker versions (6), (7). Since $S_{+-} \geq 0$ the bound (5) is not useful in this case. We see that the bounds (3) and (4) are equal at some value of $\cos\theta$; at smaller $\cos\theta$

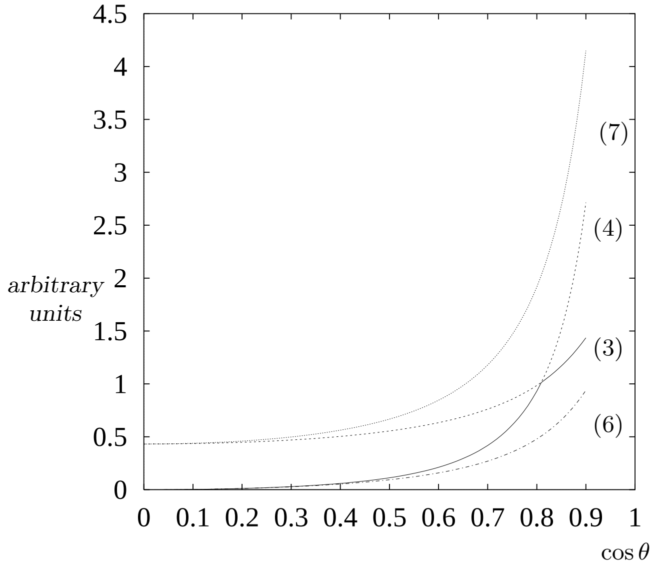

Fig. 3. Bounds (3), (4), (6), (7) on the longitudinal cross section $S_{00} = d\sigma_{00}/(dx_{I\!P}\, d\cos\theta)$ for a final state $X = q\bar{q}$, calculated by two-gluon exchange [5]. The values of the relevant parameters are $Q^2 = 45$ GeV^2, $\beta = 0.9$, $\varepsilon = 0.8$

the curve for S_{00} coincides with the upper bound and at larger $\cos\theta$ with the lower one. This crossover happens at $\cos^2\theta \approx \frac{(2\beta-1)^2}{1+4\,\beta(1-\beta)(1/\varepsilon-1)}$ if the corresponding value of P_T^2 is above a few GeV^2 so that certain approximations of S_{mn} are valid. We also find that bound (6) is rather close to (3) and (7) to (4) at small $\cos\theta$, while at the crossover point the ratio of (6) to (3) and of (4) to (7) is easily found to be 1/2. We remark that the lower bounds go to zero at $\cos\theta \to 0$ because for any final state the interference term S_{+0} vanishes at $\theta = \pi/2$ due to symmetry reasons [5]. The curves in Fig. 3 stop at very large $\cos\theta$ where the approximation used in their calculation becomes inaccurate. From an experimental point of view it should be difficult to measure φ if θ is below some critical value, this implies that an upper bound can only be given for F_L^D in a restricted kinematical region, unless one is willing to extrapolate a measured upper bound on $d\sigma_{00}/(dx_{I\!P}\, d\cos\theta)$ down to $\theta = 0$.

It is worthwhile noting that the transverse-transverse interference term S_{+-} for $q\bar{q}$ is found to be positive, whereas for the production of a $\pi^+\pi^-$ pair we have seen in the previous subsection that $S_{+-} = -S_{++} \leq 0$. In other words the preferred orientation of a quark-antiquark pair is perpendicular to the electron plane in the $\gamma^* p$ c.m. while a pair of pions prefers to be in that plane. Parton-hadron duality has recently been invoked in [9] to calculate the production of $\pi\pi$ from $q\bar{q}$ in the region of low-lying resonances like the ρ. If one takes this idea literally then the change of the azimuthal distribution from $q\bar{q}$ to $\pi\pi$ is an interesting effect of hadronisation—beyond the change in the θ distribution imposed by angular momentum conservation.[2] This also implies that a parton level calculation for the angular distribution cannot be used if the multiplicity of X is too small.

4.3 $X = q\bar{q}g$

For the final state with a $q\bar{q}$ pair and an additional gluon no complete calculation with two-gluon exchange has been performed yet. Results in the leading $\alpha_s \log Q^2$ approximation have e.g. been reported in [10]: the transverse structure function F_T^D for $q\bar{q}g$ behaves like $(1-\beta)^3$ at large β and is negligible compared with the $q\bar{q}$ contribution for $\beta > 1/2$, while F_L^D for $q\bar{q}g$ is zero in this approximation.

The three parton final state has also been investigated in the semiclassical model of [11] who find that it gives leading twist contributions both to F_T^D and F_L^D. In [12] it was shown that this approach can be reformulated in terms of the diffractive parton model: the proton emits a parton which scatters on the γ^*, producing two of the partons in X. It is required that their transverse momentum in the $\gamma^* p$ frame be sufficiently large for this scattering to be hard. The third parton in X is approximately collinear with the proton and plays the role of a "pomeron remnant".

Let us then take a closer look at what the parton model description gives for the longitudinal cross section and for the $\gamma^* p$ interference terms with the final states just described. The calculation is completely analogous to the one for the azimuthal dependence in nondiffractive deep inelastic scattering with two partons and a proton remnant in the final state, which can e.g. be found in [13].

Call the four-momenta of the two partons produced in the hard scattering P_1 and P_2, with P_1 being the forward particle, i.e. having the larger longitudinal momentum along the photon direction in the c.m. of X. Let further be P_T the transverse momentum of P_1 in that frame and $\hat{s} = (P_1 + P_2)^2$. We first give $\gamma^* p$ interference terms defined with respect to the azimuthal angle φ' between the electron momentum k and the vector $\tau' = (P_1 - P_2)/\hat{s}$. Restricting our analysis to sufficiently large P_T the effects of a nonzero transverse momentum of the parton emitted by the proton (and thus of the pomeron remnant) should not be too large and we set this transverse momentum to zero.[3] We then have

$$\frac{d\sigma_{mn}}{dx_{I\!P}} = \sum_q \frac{\pi\alpha_s\alpha_{em}e_q^2}{(1-x)\,Q^2} \int_\beta^{\hat{\beta}_{max}} d\hat{\beta} \int_{P^2_{min}}^{\hat{s}/4} \frac{dP_T^2}{\sqrt{\hat{s}(\hat{s}/4 - P_T^2)}} \cdot \frac{\beta}{\hat{\beta}} \cdot \left[g\left(\frac{\beta}{\hat{\beta}}, x_{I\!P}\right) T_{mn}^{q\bar{q}} + q\left(\frac{\beta}{\hat{\beta}}, x_{I\!P}\right) T_{mn}^{gq} + \bar{q}\left(\frac{\beta}{\hat{\beta}}, x_{I\!P}\right) T_{mn}^{gq} \right] , \tag{11}$$

[2] At $\theta = 0$ we must have $S_{++} = 0$ for $\pi\pi$ but not for $q\bar{q}$ and $S_{00} = 0$ for $q\bar{q}$ but not for $\pi\pi$, assuming again that q and $\bar{q}$ are produced with opposite helicities

[3] Intrinsic transverse parton momentum in nondiffractive ep scattering has been investigated in [14]

where $\hat\beta = Q^2/(Q^2+\hat s)$ and its upper limit $\hat\beta_{max}$ follows from the lower cutoff on P_T. The sum $\sum_q$ is over the flavours of the quarks and e_q denotes their charge in units of the positron charge. For simplicity we have taken the quarks to be massless, neglecting the complications for charm production. $g(z,x_{I\!P})$, $q(z,x_{I\!P})$ and $\bar q(z,x_{I\!P})$ respectively are the diffractive gluon, quark and antiquark distributions for a momentum fraction z of the parton with respect to the momentum transfer Δ from the proton. They are integrated over $t=\Delta^2$, so that to leading order in α_s one has $F_2^{D(3)}(x_{I\!P},\beta,Q^2) = \sum_q e_q^2\,\beta\left(q(\beta,x_{I\!P}) + \bar q(\beta,x_{I\!P})\right)$. Finally we have

$$
\begin{aligned}
T^{q\bar q}_{++} &= \frac{1}{2}\cdot 8\left(1-2\hat\beta(1-\hat\beta)\right)\left[\frac{\hat s/4-P_T^2}{P_T^2}+\frac{1}{2}\right]\\
T^{q\bar q}_{00} &= \frac{1}{2}\cdot 16\,\hat\beta(1-\hat\beta)\\
T^{q\bar q}_{+-} &= -\frac{T^{q\bar q}_{00}}{2}\\
T^{q\bar q}_{+0} &= \frac{1}{2}\cdot\frac{8}{\sqrt2}\,\frac{\sqrt{\hat s/4-P_T^2}}{P_T}\,\sqrt{\hat\beta(1-\hat\beta)}\,(2\,\hat\beta-1) \qquad (12)
\end{aligned}
$$

for boson-gluon fusion $\gamma^* g\to q\bar q$ and

$$
\begin{aligned}
T^{gq}_{++} &= \frac{4}{3}\cdot\frac{1}{1-\hat\beta}\Big[4\,(1+\hat\beta^2)\,\frac{\hat s/4-P_T^2}{P_T^2}\\
&\quad +5-2\,\hat\beta(1-\hat\beta)\Big]\\
T^{gq}_{00} &= \frac{4}{3}\cdot 4\,\hat\beta\\
T^{gq}_{+-} &= -\frac{T^{gq}_{00}}{2}\\
T^{gq}_{+0} &= \frac{4}{3}\cdot\frac{4}{\sqrt2}\,\frac{\sqrt{\hat s/4-P_T^2}}{P_T}\,\frac{\sqrt{\hat\beta^3}}{\sqrt{1-\hat\beta}} \qquad (13)
\end{aligned}
$$

for the QCD Compton processes $\gamma^* q\to gq$ and $\gamma^*\bar q\to g\bar q$. We see that at small $P_T^2/\hat s$ the $\gamma^* p$ cross sections and interference terms have the same relative factors of $1/P_T$ as in the case $X=q\bar q$ so that in this region the transverse cross section dominates. The absolute behaviour in P_T is however different; integrating over P_T one finds that $d\sigma_{00}/dx_{I\!P}$, $d\sigma_{+-}/dx_{I\!P}$ and $d\sigma_{+0}/dx_{I\!P}$ behave like $1/Q^2$ at fixed β and $x_{I\!P}$, corresponding to leading twist contributions to the ep cross section, whereas $d\sigma_{++}/dx_{I\!P}$ has a collinear singularity at $P_T=0$ which with an appropriate cutoff gives a leading twist contribution enhanced by $\log Q^2$, as it is also found in the two-gluon exchange calculation [10].

Looking at the region of large β we see in (12), (13) that the longitudinal cross section is suppressed compared with the transverse one by a factor $(1-\hat\beta)\le(1-\beta)$, both for boson-gluon fusion and QCD Compton scattering. The behaviour of F_L^D in the large-β limit depends on how the diffractive parton distributions behave for $z\to 1$. If one assumes a power behaviour $g(z,x_{I\!P})\sim(1-z)^{n_g}$ and $q(z,x_{I\!P})$, $\bar q(z,x_{I\!P})\sim(1-z)^{n_q}$ with exponents $n_g,n_q>-1$ then F_L^D is bounded from above by $c_g\,(1-\beta)^{n_g+2}+c_q\,(1-\beta)^{n_q+1}$ with some constants c_g, c_q. It was argued in [15] that the behaviour of the parton distributions should be between $(1-z)^0$ and $(1-z)^1$ for gluons and between $(1-z)^1$ and $(1-z)^2$ for quarks; in this case F_L^D would vanish at least like $(1-\beta)^2$. In [16] the ratio F_L^D/F_T^D was calculated in the diffractive parton model with a particular ansatz for the parton distributions and indeed came out small for $\beta\ge 1/2$; it is on the contrary at small β where this ratio was found to be appreciable.

Let us now investigate the bounds one can obtain for S_{00}, first for the differential quantities $S_{mn}=d\sigma_{mn}/(dx_{I\!P}\,dP_T^2\,d\hat\beta)$. From (11) to (13) one can show that for all values of the kinematic variables the expansion of the square roots in (3), (4) which leads to the simplified bounds (6), (7) is an approximation better than 5% so that it is enough to discuss the latter. Unlike in the case $X=q\bar q$ one now finds that (8) is always a strict inequality; in fact already the summation over particle helicities in the diffractive final state violates the conditions for equality in (8). It turns out that now $4\,|S_{+0}|^2\le S_{00}\,(S_{++}-S_{+-})$ with a factor 4 instead of 2 on the l.h.s. The ratio of right and left hand side goes to 1 if $P_T\to 0$ and $\hat\beta=1$ for QCD Compton scattering and if $P_T\to 0$ and $\hat\beta=1$ or $\hat\beta=0$ for boson-gluon fusion. With this we have that the lower bound (6) is at most $0.5\cdot S_{00}$. To give a numerical example away from the edges of phase space we take $4P_T^2/\hat s\ge 0.2$, $0.1\le\hat\beta\le 0.9$ and $\varepsilon=0.8$ and find that the bound is between 0 and $0.33\cdot S_{00}$ for boson-gluon fusion and between 0 and $0.38\cdot S_{00}$ for QCD Compton scattering.

The upper bound (5) is now better than (7) because $S_{+-}\le 0$, and becomes good where $S_{++}+S_{+-}$ is not large compared to S_{00}. From (12), (13) we see that this is only the case if $4P_T^2/\hat s$ is large enough. Comparing $T^{q\bar q}_{++}+T^{q\bar q}_{+-}\ge\frac12\cdot 4\,(2\hat\beta-1)^2$ with $T^{q\bar q}_{00}$ and $T^{gq}_{++}+T^{gq}_{+-}\ge\frac43\cdot(1-\hat\beta)^{-1}$ with T^{gq}_{00} we further see that $\hat\beta(1-\hat\beta)$ must not be small. For $\varepsilon=0.8$, $0.2\le\hat\beta\le 0.8$ and $4P_T^2/\hat s=0.5$ we find an upper bound between $2.2\cdot S_{00}$ and $4.4\cdot S_{00}$ for boson-gluon fusion and between $12\cdot S_{00}$ and $22.5\cdot S_{00}$ for the QCD Compton process, when going down to $4P_T^2/\hat s=0.1$ these bounds become about five times larger.

We now have to see how the interference terms corresponding to the vector τ defined from the thrust axis are related to those discussed so far. We recall that for a system $X=q\bar q g$ with zero quark mass the thrust axis in its rest frame is given by the direction of the most energetic particle. This can be (i) the forward parton produced in the hard γ^* parton collision or (ii) the parton playing the role of a pomeron remnant. For events of type (i) we have $\varphi=\varphi'$, i.e. τ and τ' lead to the same $\gamma^* p$ interference terms given in (11) to (13). In our simple calculation with zero transverse momentum for the pomeron remnant events of type (ii) have τ collinear with q and p and do not contribute to the φ-asymmetry in the ep cross sections, the corresponding interference terms thus are zero [5]. The condition for (i) is $1-\sqrt{1-4P_T^2/\hat s}<2\beta(1-\hat\beta)/(\hat\beta-\beta)$.

It is always fulfilled for $\hat{\beta} < 3\beta/(2\beta+1)$ and otherwise only for P_T below some critical value.

Unless one attempts a separation of final states $q\bar{q}$ and $q\bar{q}g$, using for instance the value of the thrust, one will sum over them when evaluating the S_{mn}. To investigate their relative importance is beyond the scope of our study, but our arguments have shown that in regions of phase space where $q\bar{q}g$ states dominate the interference terms and the longitudinal cross section the bounds will not be very tight, whereas quite good bounds can be expected where the $q\bar{q}$ state dominates.

Beyond the possibility to obtain bounds on F_L^D the $\gamma^* p$ interference terms are interesting in themselves. From (11) and (12), (13) we see that both for boson-gluon fusion and QCD Compton scattering the transverse-transverse interference is negative and thus has the opposite sign than what we found for $X = q\bar{q}$ (cf. also [8]). The transverse-longitudinal interference is more complicated, but if $\beta > 1/2$ one has $2\hat{\beta} - 1 > 0$ and it is positive for $X = q\bar{q}g$ in our parton model calculation and thus again opposite to the one for $X = q\bar{q}$ calculated by two-gluon exchange. In this sense the sign of the interference terms gives a hint on the underlying final state and its production mechanism.

Another difference between the final states is the leading power behaviour in $1/Q$ at fixed β and $x_{I\!P}$ of the integrated interference terms and cross sections $S_{mn} = d\sigma_{mn}/dx_{I\!P}$. For $X = q\bar{q}$ we found $S_{+-} \sim 1/Q^4$ and $S_{+0} \sim 1/Q^3$ compared with $S_\varepsilon \sim 1/Q^2$ whereas for $X = q\bar{q}g$ all three terms go like $1/Q^2$. There will be logarithmic corrections to these powers, but unless they strongly differ for S_{+-}, S_{+0} and S_ε the relative behaviour of the interference terms and the sum of cross sections is clearly distinct in the two cases. More generally the inequality (8) connects the Q^2 dependence of S_{00} and S_{+0}: an interference S_{+0} that only decreases like $1/Q^2$ excludes a nonleading twist behaviour of S_{00} beyond some value of Q^2 if we assume that $S_{++} - S_{+-}$ is leading twist (experiment indicates that S_ε is).

5 Conclusions

The distribution of an azimuthal angle defined with the help of the thrust axis of the diffractive final state allows to extract interference terms between different polarisations of the exchanged photon in diffractive deep inelastic scattering. They may help to answer the important question of whether the cross section for longitudinal photons is leading twist or not and furthermore give information on which diffractive final states dominate in a given kinematic region.

These interference terms can be used to obtain model independent bounds on the longitudinal cross section. We have shown that it can be advantageous to evaluate these bounds first with some additional binning in variables like the polar angle θ of the thrust axis. Such differential bounds can be equal to the longitudinal cross section itself. For diffractive final states $\pi\pi$ or KK this happens under weak dynamical assumptions, which should be satisfied to a good approximation in the diffractive regime.

Using the results of two-gluon exchange models one has that the $q\bar{q}$ diffractive final state gives a longitudinal contribution F_L^D to F_2^D which is suppressed by $1/Q^2$ but can be non-negligible at large β. The estimated bounds one could obtain on F_L^D in this kinematic region look quite good, especially if one evaluates them first binned in θ. To investigate $q\bar{q}g$ final states we used the diffractive parton model. We have shown that one does not expect these final states to lead to an appreciable ratio F_L^D/F_T^D at large β, but remark that a ratio of up to 0.5 was found at small β in [16]. In an estimation neglecting the effects of intrinsic parton momentum and hadronisation, which should be valid if there is large enough P_T in the diffractive system we find that if this final state dominates then the bounds on F_L^D obtained from the interference terms are much less stringent than in the $q\bar{q}$ case, except in some corners of phase space.

Acknowledgements. I gratefully acknowledge conversations with J. Bartels, A. Hebecker, M. McDermott and M. Wüsthoff. CPT is Unité Propre 14 du Centre National de la Recherche Scientifique.

References

1. H1 Collaboration, "Inclusive Measurement of Diffractive Deep-inelastic *ep* Scattering", paper submitted to the International Europhysics Conference on High Energy Physics, HEP97, Jerusalem, Israel, August 1997, preprint DESY–97–158
2. G. Briskin and M. F. McDermott, "Diffractive structure functions in DIS", in: Future Physics at HERA, Proc. of the Workshop 1995/96, eds. G. Ingelman et al., DESY 1996
3. J. Bartels, talk given at the 5th International Workshop on Deep Inelastic Scattering and QCD (DIS 97), Chicago, IL, 14–18 April 1997, to appear in the proceedings; talk given at the Madrid Workshop on low-x Physics, Miraflores de la Sierra, Spain, 18–21 June 1997, to appear in the proceedings
4. T. Arens, O. Nachtmann, M. Diehl and P. V. Landshoff, Z. Phys. C **74** (1997) 651
5. M. Diehl, Z. Phys. C **76** (1997) 499
6. P. V. Landshoff and O. Nachtmann, Z. Phys. C **35** (1987) 405
7. A. Donnachie and P. V. Landshoff, Nucl. Phys. B **244** (1984) 322
8. J. Bartels, C. Ewerz, H. Lotter and M. Wüsthoff, Phys. Lett. B **386** (1996) 389
9. A. D. Martin, M. G. Ryskin, and T. Teubner, Phys. Rev. D **55** (1997) 4329; Phys. Rev. D **56** (1997) 3007
10. M. Wüsthoff, Phys. Rev. D **56** (1997) 4311
11. W. Buchmüller, M. F. McDermott and A. Hebecker, Nucl. Phys. B **487** (1997) 283
12. A. Hebecker, Nucl. Phys. B **505** (1997) 349
13. H. Georgi and J. Sheiman, Phys. Rev. D **20** (1979) 111
14. A. König and P. Kroll, Z. Phys. C **16** (1982) 89;
A. S. Joshipura and G. Kramer, J. Phys. G **8** (1982) 209
15. A. Berera and D. E. Soper, Phys. Rev. D **53** (1996) 6162
16. K. Golec-Biernat and J. Kwieciński, Phys. Lett. B **353** (1995) 329